Reply to the Comment of R. M. Cavalcanti on "Resonant Spectra and the Time Evolution of the Survival and Nonescape Probabilities"

In our paper [1] we derived an exact expression for the nonescape probability $P(t)$, (see Eq. (14)), as an expansion in terms of resonant states and M functions,

$$P(t) = \sum_{n=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} C_n C_\ell^* I_{n\ell} M(k_n, t) M^*(k_\ell, t), \quad (1)$$

where the integral $I_{n\ell}$ is defined by Eq. (15) of ref. [1],

$$I_{n\ell} = \int_0^R u_\ell^*(r) u_n(r) dr. \quad (2)$$

The long time limit of $P(t)$ leads to an asymptotic expansion whose leading term reads,

$$\sum_{n=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} \left(\frac{C_n C_\ell^* I_{n\ell}}{k_n k_\ell^*} \right) \frac{1}{t}. \quad (3)$$

So we concluded that at long times $P(t) \sim t^{-1}$. Cavalcanti [2] instead has proven that the above coefficient vanishes and concludes that the leading term of $P(t) \sim t^{-3}$. His procedure corresponds to interchange the integral over r in the expression of $P(t)$, Eq. (1), with the long time limit. The vanishing of the term proportional to t^{-1} then follows from the sum rule

$$\sum_{m=-\infty}^{\infty} \frac{C_m u_m(r)}{k_m} = 0, \quad (r \leq R). \quad (4)$$

In our approach we perform first the integration over r and then take the long time limit. We provide below an argument that shows that in dealing with resonant state expansions the interchange of the above operations do not lead to the same result. This is the case for expansions that do not converge uniformly.

Consider the *n-th* resonant function $u_n(r)$ is a solution of the Schrödinger equation [3],

$$u_n^{''}(r) + [k_n^2 - V(r)] u_n(r) = 0, \quad (5)$$

where the prime stands for the derivative with repect to r, k_n^2 is a squared complex wavenumber, and $V(r)$ is an arbitrary potential that vanishes beyond $r = R$. The function $u_n(r)$ satisfies the boundary conditions,

$$u_n(0) = 0; \quad \Big[u_n^{'}(r) \Big]_{r=R} = i k_n u_n(R). \quad (6)$$

Consider also similar equations for the complex $\ell - th$ function $u_\ell^*(r)$,

$$u_\ell^{''*}(r) + [k_\ell^{*2} - V(r)] u_\ell^*(r) = 0, \quad (7)$$

which obeys the boundary conditions,

$$u_\ell^*(0) = 0; \quad \Big[u_\ell^{'*}(r) \Big]_{r=R} = -i k_\ell^* u_\ell^*(R). \quad (8)$$

Now multiply Eq. (5) by $u_\ell^*(r)$ and substract from it Eq. (7) multiplied by $u_n(r)$. Integrating the resulting expression from $r = 0$ to $r = R$ yields,

$$\Big[u_n(r) u_\ell^{'*}(r) - u_\ell^*(r) u_n^{'}(r) \Big]_{r=0}^{r=R} + (k_n^2 - k_\ell^{*2}) I_{n\ell} = 0. \quad (9)$$

Using Eqs. (6) and (8) allows to write a closed form of $I_{n\ell}$,

$$I_{nl} = \frac{u_n(R) u_\ell^*(R)}{i(k_n - k_\ell^*)}. \quad (10)$$

Substituting Eq. (10) into Eq. (1) leads to the following exact expression for the nonescape probability,

$$P(t) = \sum_{n=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} C_n C_\ell^* \frac{u_n(R) u_\ell^*(R)}{i(k_n - k_\ell^*)} M(k_n, t) M^*(k_\ell, t). \quad (11)$$

Taking now the long time limit allows to write $P(t)$ at leading order in inverse powers of t as,

$$P(t) \sim \sum_{n=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} \left(\frac{C_n}{k_n} \frac{C_\ell^*}{k_\ell^*} \frac{u_n(R) u_\ell^*(R)}{i(k_n - k_\ell^*)} \right) \frac{1}{t}. \quad (12)$$

The sum rule given by Eq. (4) does not lead to the vanishing of Eq. (12) because of the existence of the factor $1/(k_n - k_\ell^*)$. This shows that the interchange of integration and the long time limit operations on the resonant expansions yields different results. In our opinion, according to the definition of $P(t)$, the integration over r should precede the long time limit and consequently $P(t) \sim t^{-1}$.

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[1] G. García-Calderón, J. L. Mateos and M. Moshinsky, Phys. Rev. Lett. **74**, 337 (1995).

[2] R. M. Cavalcanti, Phys. Rev. Lett. preceding Comment.
[3] G. García-Calderón and R.E. Peierls, Nucl. Phys. **A 265**, 441 (1976); G. García-Calderón, J. L. Mateos and M. Moshinsky, Ann. Phys. (N.Y.) **249**, 430 (1996).